Some Hierarchy Results of Alternating Automata with Counters and Stack-Counters

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Some Hierarchy Results of Alternating Finite Automata with Counters and Stack-Counters

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1 Introduction

Alternating Turing machines were introduced in [2] as a mechanism to model parallel computation, and in the related papers [3–8], investigations of alternating machines have been continued. Inoue, Ito and Takanami [3,4] investigated hierarchical properties in the accepting powers of realtime one-way alternating multi-counter automata and multi-stack-counter automata. Further, Yoshinaga, Inoue and Takanami [5] strengthened the results in [3,4].

In this paper, we introduce a new machine model, a realtime one-way alternating finite automaton with counters and stack-counters, in order to investigate the essential difference between counters and stack-counters. Section 2 gives the definitions and notations necessary for this paper. Section 3 investigates a relationship between the accepting powers of realtime one-way alternating finite automata with counters and stack-counters which have only universal states, which have only existential states, and which have full alternation. For each $k \geq 0$, $l \geq 0$ $((k,l) \neq (0,0))$, let 1A(k,l,real) denote the class of sets accepted by realtime one-way alternating finite automata with k counters and l stack-counters, let 1U(k,l,real) denote the class of sets accepted by realtime one-way alternating finite automata with k counters and l stack-counters which have only universal states, and let 1N(k,l,real) denote the class of sets accepted by realtime one-way nondeterministic finite automata with k counters and l stack-counters. Further, let 1D(k,l,real) denote the class of sets accepted by realtime one-way deterministic finite automata with k counters and l stack-counters. We show that for each $k \geq 0$, $l \geq 0$ $((k,l) \neq (0,0))$ and each X$\in$\{U,N\}, 1D(k,l,real) $\subsetneq$ 1X(k,l,real) $\subsetneq$ 1A(k,l,real). Section 4 investigates hierarchical properties based on the numbers of counters and stack-counters. Book and Ginsburg [1] showed that for each $k \geq 1$ and each X$\in$\{N,D\}, 1X($0,k+1$,real) $-$ 1X($0,k$,real) $\neq \phi$. Inoue, Ito and Takanami [4] showed that for each $k \geq 1$, 1A($0,k+1$,real) $-$ 1A($0,k$,real) $\neq \phi$. Further, it is shown in [5] that for each $k \geq 1$, 1U($0,k+1$,real) $-$ 1A($0,k$,real) $\neq \phi$. We strengthen these results, and show, for example, that for each $k \geq 0$, $l \geq 0$ $((k,l) \neq (0,0))$, 1U($k+1,l$,real) $-$ 1A(k,l,real) $\neq \phi$, and 1D($k+1,l$,real) $-$ 1X(k,l,real) $\neq \phi$ for each X$\in$\{U,N\}. Section 5 investigates a relationship between the accepting powers of counters and stack-counters. Book and Ginsburg [1] showed that for each $k \geq 1$, 1N($0,k$,real) $-$ 1N($2k-1,0$,real) $\neq \phi$, and it is shown in [5] that for each $k \geq 1$, 1U($0,k$,real) $-$ 1A($k,0$,real) $\neq \phi$, and 1D($0,k$,real) $-$ 1U($2k-1,0$,real) $\neq \phi$. We show, for example, that for each $k \geq 0$, $l \geq 0$ and $m \geq 1$, 1U($k,l+m$,real) $-$ 1A($k+2m-1,l$,real) $\neq \phi$, and 1D($k,l+m$,real) $-$ 1X($k+2m-1,l$,real) $\neq \phi$ for each X$\in$\{U,N\}. Section 6 concludes this paper by giving several open problems.

2 Preliminaries

A one-way multi-counter automaton is a one-way multi-pushdown automaton whose pushdown stores operate as counters, i.e., each storage tape is a pushdown tape of the form Z^i (Z fixed). (See [1,9,10] for formal definitions of one-way multi-counter automata.) A one-way multi-stack-counter automaton is a one-way multi-counter automaton with the added property that each counter may be entered without erasing. In addition, the automaton has the ability to sense the leftmost and rightmost symbols on each stack. We assume that the rightmost symbol on each stack-counter is the top symbol on the stack. (See [1] for formal definitions of one-way multi-stack-counter automata.)

A one-way alternating multi-counter automaton (1amca) (resp., a one-way alternating multi-stack-counter automaton (1amsca)) M is the generalization of a one-way nondeterministic multi-counter automaton (resp., a one-way nondeterministic multi-stack-counter automaton) in the same sense as in [2,6,7]. That is, the state set of M is divided into two disjoint sets, the set of *universal* states and the set of *existential* states. Of course, M has a specified set of *accepting* states.

Here, we introduce a new machine model, a *one-way alternating finite automaton with counters and stack-counters* (1afacs) M, in order to investigate the essential difference between counters and stack-counters. That is, M has counters and stack-counters, and is a generalization of 1amca and 1amsca.

For each $k \geq 0$, $l \geq 0$ $((k,l) \neq (0,0))$, we denote a one-way alternating finite automaton with k counters and l stack-counters by 1afacs(k,l).

We assume that 1afacs's have the right endmarker "\$" on the input tape, read the input tape from left to right, and can enter an accepting state only when falling off the right endmarker \$. We also assume that in one step 1afacs's can increment or decrement the contents (i.e., the length) of each counter and stack-counter by at most one.

An *instantaneous description* (ID) of 1afacs(k,l) M is an element of

$$\Sigma^* \times N \times S_M,$$

where Σ ($\$ \notin \Sigma$) is the input alphabet of M, N denotes the set of all positive integers, and $S_M = Q \times (Z^*)^k \times (Z^*)^l \times (N \cup \{0\})^l$ (where Q is the set of states of the finite control of M, and Z is the storage symbol of M). The first and second components, w and i, of an ID $I = (w, i, (q, (\alpha_1, \ldots, \alpha_k), (\beta_1, \ldots, \beta_l), (j_1, \ldots, j_l)))$ represent the input string and the input head position, respectively.[1] The third component $(q,(\alpha_1,\dots,\alpha_k),(\beta_1,\dots,\beta_l),(j_1,\dots,j_l))$ of I represents the state of the finite control, the contents of the k counters, the contents of the l stack-counters, and the positions of the l stack-counter heads. (These positions are counted from left to right.) For each j $(1 \le j \le k)$, α_j is called a *storage state of the j-th counter*, and for each i $(1 \le i \le l)$, the pair (β_i, j_i) is called a *storage state of the i-th stack-counter*. An element of S_M is called a *storage state* of M. If q is the state associated with an ID I, then I is said to be a *universal* (*existential, accepting*) ID if q is a universal (existential, accepting) state. The *initial* ID of 1afacs(k,l) M on $w \in \Sigma^*$ is $I_M(w) = (w, 1, (q_0, (\lambda,\dots,\lambda), (\lambda,\dots,\lambda), (0,\dots,0)))$, where q_0 is the initial state of M and λ denotes the empty string.

We write $I \vdash_M I'$ and say I' is a *successor* of I if an ID I' follows from an ID I in one step, according to the transition function of M. A *computation path* of M on input w is a sequence $I_0 \vdash_M I_1 \vdash_M \dots \vdash_M I_n$ $(n \ge 0)$, where $I_0 = I_M(w)$. A *computation tree* of M is a finite, nonempty labeled tree with the following properties:

1. each node π of the tree is labeled with an ID, $\ell(\pi)$,
2. if π is an internal node (a non-leaf) of the tree, $\ell(\pi)$ is universal and $\{I|\ell(\pi) \vdash_M I\} = \{I_1, I_2, \dots, I_r\}$, then π has exactly r children $\rho_1, \rho_2, \dots, \rho_r$ such that $\ell(\rho_i) = I_i$, and
3. if π is an internal node of the tree and $\ell(\pi)$ is existential, then π has exactly one child ρ such that $\ell(\pi) \vdash_M \ell(\rho)$.

A *computation tree of M on input w* is a computation tree of M whose root is labeled with $I_M(w)$. An *accepting* computation tree of M on w is a computation tree of M on w whose leaves are all labeled with accepting ID's. We say that M *accepts* w if there is an accepting computation tree of M on w. We denote the set of input words accepted by M by $T(M)$.

For each $k \ge 0$, $l \ge 0$ $((k,l) \ne (0,0))$, let 1ufacs(k,l) denote a 1afacs(k,l) with only universal states, and let 1nfacs(k,l) denote a one-way nondeterministic finite automaton with k counters and l stack-counters, that is, a 1afacs(k,l) which has no universal states. Further, let 1dfacs(k,l) denote a one-way deterministic finite automaton with k counters and l stack-counters.

For each x∈{a,u,n,d}, a 1xfacs(k,l) M *operates in time* $T(n)$ if for each input w accepted by M, there is an accepting computation tree of M on w such that the length of each computation path of the tree is at most $T(|w|)$. M operates in *realtime* if $T(n) = n+1$. For each x∈{a,u,n,d}, we denote by 1xfacs$(k,l,$real$)$ a 1xfacs(k,l) which operates in realtime. We define

$$\begin{aligned}
1A(k,l,\text{real}) &= \{L|L = T(M) \text{ for some 1afacs}(k,l,\text{real})\ M\},\\
1U(k,l,\text{real}) &= \{L|L = T(M) \text{ for some 1ufacs}(k,l,\text{real})\ M\},\\
1N(k,l,\text{real}) &= \{L|L = T(M) \text{ for some 1nfacs}(k,l,\text{real})\ M\}, \text{ and}\\
1D(k,l,\text{real}) &= \{L|L = T(M) \text{ for some 1dfacs}(k,l,\text{real})\ M\}.
\end{aligned}$$

It is shown in [1] that for each $k \ge 1$, each one-way nondeterministic k-stack-counter automaton can be simulated by one-way nondeterministic $2k$-counter automaton without loss of time. By using the same technique as in the proof of this fact, we can easily show that the following fact holds.

Fact 2.1. For each $k \ge 0$, $l \ge 0$ and $m \ge 0$ $((k,l,m) \ne (0,0,0))$, and each X∈{A,U,N,D},
1X$(k, l+m,$real$) \subseteq$ 1X$(k+2m, l,$real$)$.

3 A Relationship between 1D$(k,l,$real$)$, 1N$(k,l,$real$)$, 1U$(k,l,$real$)$ and 1A$(k,l,$real$)$

This section investigates a relationship between the accepting powers of realtime 1afacs's with only universal states, with only existential states, and with full alternation. Now, let

$$\begin{aligned}
1N(k,l,T(n)) &= \{L|L = T(M) \text{ for some 1nfacs}(k,l)\ M \text{ operating in } T(n)\}, \text{ and}\\
1U(k,l,T(n)) &= \{L|L = T(M) \text{ for some 1ufacs}(k,l)\ M \text{ operating in } T(n)\}.
\end{aligned}$$

Yoshinaga, Inoue and Takanami [5] showed that for each $k \ge 1$ and each X∈{U,N},

$$1D(k,0,\text{real}) \subsetneq 1X(k,0,\text{real}) \subsetneq 1A(k,0,\text{real}),\ 1D(0,k,\text{real}) \subsetneq 1X(0,k,\text{real}) \subsetneq 1A(0,k,\text{real}),$$
$$1U(k,0,\text{real}) \text{ is incomparable with } 1N(k,0,\text{real}), \text{ and } 1U(0,k,\text{real}) \text{ is incomparable with } 1N(0,k,\text{real}).$$

We below show that a similar result holds for 1afacs's. From Lemma 3.1 in [5], we can easily show the following result.

Lemma 3.1. Let $L_1 = \{wcw|w \in \{0,1\}^+\}$, and $L_2 = \{wcw'|w \in \{0,1\}^+, w' \ne w\}$. Then,
(1) $L_1 \in$ 1U(1,0,real),
(2) $L_2 \in$ 1N(1,0,real),
(3) $L_2 \notin \bigcup_{1 \le k < \infty} \bigcup_{1 \le l < \infty} \bigcup_{1 \le r < \infty} 1U(k,l,n^r)$, and
(4) $L_1 \notin \bigcup_{1 \le k < \infty} \bigcup_{1 \le l < \infty} \bigcup_{1 \le r < \infty} 1N(k,l,n^r)$.

From Lemma 3.1 above, we have the follwing results.

[1] We note that $1 \le i \le |w|+2$, where for any strings v, $|v|$ denotes the length of v, "1", "$|w|+1$" and "$|w|+2$" represent the positions of the leftmost symbol of w, the right endmarker \$, and the immediate right to \$. Further, we note that $0 \le j_i \le |\beta_i|$ for each $1 \le i \le l$.

Theorem 3.1. For each $k \geq 0$, $l \geq 0$ $((k,l) \neq (0,0))$,
(1) $1\mathrm{D}(k,l,\mathrm{real}) \subsetneq 1\mathrm{U}(k,l,\mathrm{real}) \subsetneq 1\mathrm{A}(k,l,\mathrm{real})$, and
(2) $1\mathrm{D}(k,l,\mathrm{real}) \subsetneq 1\mathrm{N}(k,l,\mathrm{real}) \subsetneq 1\mathrm{A}(k,l,\mathrm{real})$.

Theorem 3.2. For each $k \geq 0$, $l \geq 0$ $((k,l) \neq (0,0))$, $1\mathrm{U}(k,l,\mathrm{real})$ is incomparable with $1\mathrm{N}(k,l,\mathrm{real})$.

4 Hierarchy Results Based on the Numbers of Counters and Stack-Counters

Inoue, Ito and Takanami [3,4] showed that for each $k \geq 1$,

$$1\mathrm{A}(k+1,0,\mathrm{real}) - 1\mathrm{A}(k,0,\mathrm{real}) \neq \phi, \text{ and } 1\mathrm{A}(0,k+1,\mathrm{real}) - 1\mathrm{A}(0,k,\mathrm{real}) \neq \phi.$$

This section first shows that for each $k \geq 0$, $l \geq 0$ $((k,l) \neq (0,0))$,

$$1\mathrm{U}(k+1,l,\mathrm{real}) - 1\mathrm{A}(k,l,\mathrm{real}) \neq \phi, \text{ and } 1\mathrm{N}(k+1,l+1,\mathrm{real}) - 1\mathrm{A}(k,l,\mathrm{real}) \neq \phi.$$

This result strengthens the results in [5]

$$1\mathrm{U}(k+1,0,\mathrm{real}) - 1\mathrm{A}(k,0,\mathrm{real}) \neq \phi,\ 1\mathrm{U}(0,k+1,\mathrm{real}) - 1\mathrm{A}(0,k,\mathrm{real}) \neq \phi,$$
$$1\mathrm{N}(k+3,0,\mathrm{real}) - 1\mathrm{A}(k,0,\mathrm{real}) \neq \phi, \text{ and } 1\mathrm{N}(0,k+2,\mathrm{real}) - 1\mathrm{A}(0,k,\mathrm{real}) \neq \phi$$

for each $k \geq 1$.

To prove these results, we first give some necessary definitions. Let M be a 1afacs(k,l,real), $k \geq 0$, $l \geq 0$ $((k,l) \neq (0,0))$, and Σ be the input alphabet of M. For each storage state s of M and for each $w \in \Sigma^+$, let an *s-computation tree* of M on w is a computation tree of M whose root is labeled with the ID $(w,1,s)$. (That is, an s-computation tree of M on w is a computation tree which represents a computation of M on $w\$$ starting with the input head on the leftmost position of w and with the storage state s.) An *s-accepting computation tree* of M on w is an s-computation tree of M on w whose leaves are all labeled with accepting ID's.

For each $n \geq 1$ and for integers $a_1, a_2, \ldots, a_k$ such that $0 \leq a_j \leq n$ $(1 \leq j \leq k)$, let $p_n(a_k, a_{k-1}, \ldots, a_1)$ denote the integer represented by $(n+1)$-ary number $a_k a_{k-1} \ldots a_2 a_1$. That is,
$p_n(a_k, a_{k-1}, \ldots, a_1) = a_k \times (n+1)^{k-1} + a_{k-1} \times (n+1)^{k-2} + \ldots + a_2 \times (n+1)^1 + a_1 \times (n+1)^0$.
Let $g : (N \cup \{0\}) \times (N \cup \{0\}) \times \{0,1\} \rightarrow (N \cup \{0\})$ be the partial function such that

$$g(n,j,m) = \begin{cases} 2(\sum_{i=0}^{n} i) + n - j & \text{if } m = 0 \\ 2(\sum_{i=0}^{n} i) + n + j + 1 & \text{if } m = 1, \end{cases}$$

where $j \leq n$. If $j > n$ then $g(n,j,m)$ is undefined.

For each $n \geq 1$ and for integers $b_1, b_2, \ldots, b_l$ such that $0 \leq b_i \leq g(n,n,1)$ $(1 \leq i \leq l)$, let $q_n(b_l, b_{l-1}, \ldots, b_1)$ denote the integer represented by $(g(n,n,1)+1)$-ary number $b_l b_{l-1} \ldots b_2 b_1$. That is,
$q_n(b_l, b_{l-1}, \ldots, b_1) = b_l \times (g(n,n,1)+1)^{l-1} + b_{l-1} \times (g(n,n,1)+1)^{l-2} + \ldots + b_1 \times (g(n,n,1)+1)^0$.
Then, for each $n \geq 1$, and for integers $0 \leq a_j \leq n$ $(1 \leq j \leq k)$ and $0 \leq b_i \leq g(n,n,1)$ $(1 \leq i \leq l)$, let

$$o_n(p_n(a_k, \ldots, a_1), q_n(b_l, \ldots, b_1)) = p_n(a_k, \ldots, a_1) \times (g(n,n,1)+1)^l + q_n(b_l, \ldots, b_1).$$

The following lemma leads to our main results.

Lemma 4.1. For each $k \geq 0$, $l \geq 0$ $((k,l) \neq (0,0))$, let
$A(k,l) = \{\sharp^n 1^{s_1} \natural \ldots \natural 1^{s_k} a^{t_1} b^{u_1} c_1 \ldots a^{t_l} b^{u_l} c_l h(n,m_1) \natural \ldots \natural h(n,m_r) | n \geq 1 \;\&\; r \geq 1 \;\&\; \forall j(1 \leq j \leq k)[0 \leq s_j \leq n] \;\&\forall i(1 \leq i \leq l)[0 \leq t_i \leq n, 0 \leq u_i \leq n - t_i, c_i \in \{0,1\}] \;\&\; \forall f(1 \leq f \leq r)[m_f \geq 1] \;\&\; \exists e(1 \leq e \leq r)[m_e = o_n(p_n(n-s_k, \ldots, n-s_1), q_n(g(n-t_l, n-t_l-u_l, c_l), \ldots, g(n-t_1, n-t_1-u_1, c_1)))]\}$, and
$A'(k,l) = \{\sharp^n 1^{s_1} \natural \ldots \natural 1^{s_k} a^{t_1} b^{u_1} c_1 \ldots a^{t_l} b^{u_l} c_l h(n,m_1) \natural \ldots \natural h(n,m_r) | n \geq 1 \;\&\; r \geq 1 \;\&\; \forall j(1 \leq j \leq k)[0 \leq s_j \leq n] \;\&\; \forall i(1 \leq i \leq l)[0 \leq t_i \leq n, 0 \leq u_i \leq n - t_i, c_i \in \{0,1\}] \;\&\; \forall e(1 \leq e \leq r)[m_e \geq 1 \;\&\; m_e \neq o_n(p_n(n-s_k, \ldots, n-s_1), q_n(g(n-t_l, n-t_l-u_l, c_l), \ldots, g(n-t_1, n-t_1-u_1, c_1)))]\}$,
where $h(n,m) = (0\sharp^n)^m$. Then, for each $k \geq 0$, $l \geq 0$ $((k,l) \neq (0,0))$,
(1) $A(k,l) \in 1\mathrm{A}(k,l,\mathrm{real})$,
(2) $A'(k,l) \in 1\mathrm{U}(k,l,\mathrm{real})$, and
(3) $A(k,l) \in 1\mathrm{N}(k,l+1,\mathrm{real})$,
and for each $k \geq 1$, $l \geq 0$ $((k-1,l) \neq (0,0))$
(4) $A(k,l) \notin 1\mathrm{A}(k-1,l,\mathrm{real})$, and
(5) $A'(k,l) \notin 1\mathrm{A}(k-1,l,\mathrm{real})$,
and for each $k \geq 0$, $l \geq 1$ and $m \geq 1$ $(l - m \geq 0)$,
(6) $A(k,l) \notin 1\mathrm{A}(k+2m-1,l-m,\mathrm{real})$, and
(7) $A'(k,l) \notin 1\mathrm{A}(k+2m-1,l-m,\mathrm{real})$.

The proof of (1) and (2): $A(k,l)$ (resp., $A'(k,l)$) is accepted by a 1afacs(k,l,real) (resp., 1ufacs(k,l,real)) M which acts as follows. Let $C_1,\dots,C_k$ and $SC_1,\dots,SC_l$ be the counters and the stack-counters of M, respectively, and H be the input head of M. Suppose that an input string

$w = \sharp^n 1^{s_1}\natural 1^{s_2}\natural\dots\natural 1^{s_k} a^{t_1}b^{u_1}c_1 a^{t_2}b^{u_2}c_2\dots a^{t_l}b^{u_l}c_l 0\sharp^{n_{11}}0\sharp^{n_{12}}\dots 0\sharp^{n_{1m_1}}\natural\dots\natural 0\sharp^{n_{r1}}0\sharp^{n_{r2}}\dots 0\sharp^{n_{rm_r}}\$$

(where $n \geq 1$, $r \geq 1$, $c_i \in \{0,1\}$, n_{ij}, $m_i \geq 1$) is presented to M. (Input strings in a form different from the above can easily be rejected by M.) M universally branches to check the follwing two points:

(i) whether the initial segment $\sharp^n$ is equal to every segment $\sharp^{n_{ij}}$,

(ii) whether $0 \leq s_j \leq n$ for each j $(1 \leq j \leq k)$, $0 \leq t_i \leq n$ and $0 \leq u_i \leq n - t_i$ for each i $(1 \leq i \leq l)$, and $m_e = o_n(p_n(n-s_k,\dots,n-s_1), q_n(g(n-t_l, n-t_l-u_l, c_l),\dots,g(n-t_1,n-t_1-u_1,c_1)))$ for some e $(1 \leq e \leq r)$ (resp., $m_e \neq o_n(p_n(n-s_k,\dots,n-s_1), q_n(g(n-t_l, n-t_l-u_l, c_l),\dots,g(n-t_1,n-t_1-u_1,c_1)))$ for any e $(1 \leq e \leq r)$).

(i) above can be easily checked by using one stack-counter, and (ii) above can be checked by using the following algorithm. For each counter C_j $(1 \leq j \leq k)$, we let α_j denote the storage state of C_j. For each stack-counter SC_i $(1 \leq i \leq l)$, we store the flag F_i in the finite control. The value of F_i is either 0 or 1. For each i $(1 \leq i \leq l)$, we let (β_i, j_i) denote the storage state of SC_i, and let f_i denote the value of F_i. The *counting number* of SC_i is $g(|\beta_i|, j_i, f_i)$. For each i $(1 \leq i \leq l)$, we let d_i denote the counting number of SC_i.

(a) While reading the initial segment $\sharp^n$ of w, M stores Z^n in each of k counters $C_1,\dots,C_k$ and each of l stack-counters $SC_1,\dots,SC_l$. After that, for each j $(1 \leq j \leq k)$, on the segment 1^{s_j}, M erases Z^{s_j} in C_j while reading 1^{s_j}, and for each i $(1 \leq i \leq l)$, on the segment $a^{t_i}b^{u_i}c_i$, M erases Z^{t_i} in SC_i while reading a^{t_i}, moves the i-th stack-counter head u_i cells to the left without erasing Z^{n-t_i} in SC_i while reading b^{u_i} and sets $f_i = c_i \in \{0,1\}$. During this action, M can check whether $0 \leq s_j \leq n$ for each j $(1 \leq j \leq k)$, and $0 \leq t_i \leq n$ and $0 \leq u_i \leq n - t_i$ for each i $(1 \leq i \leq l)$. When H reaches the symbol "0" just after c_l, $\alpha_j = Z^{n-s_j}$ for each $1 \leq j \leq k$, $\beta_i = Z^{n-t_i}$, $j_i = n - t_i - u_i$, $f_i = c_i$, and $d_i = g(n-t_i, n-t_i-u_i, c_i)$ for each $1 \leq i \leq l$, and thus

$o_n(p_n(|\alpha_k|,\dots,|\alpha_1|), q_n(d_l,\dots,d_1)) =$
$o_n(p_n(n-s_k,\dots,n-s_1), q_n(g(n-t_l,n-t_l-u_l,c_l),\dots,g(n-t_1,n-t_1-u_1,c_1)))$.

(b) Assuming that (i) above is successfully checked (i.e., $n = n_{ij}$ for all i, j), after reading the segment $\sharp^n 1^{s_1}\natural\dots\natural 1^{s_k}$ $a^{t_1}b^{u_1}c_1\dots a^{s_l}b^{t_l}c_l$ of the input w, M existentially guesses some e $(1 \leq e \leq r)$ and checks whether $m_e = o_n(p_n(|\alpha_k|,\dots,|\alpha_1|), q_n(d_l,\dots,d_1))$ (resp., M universally branches to check whether $m_e \neq o_n(p_n(|\alpha_k|,\dots,|\alpha_1|), q_n(d_l,\dots,d_1))$ for each ϵ $(1 \leq e \leq r)$). To check whether $m_e = o_n(p_n(|\alpha_k|,\dots,|\alpha_1|), q_n(d_l,\dots,d_1))$ (resp., $m_e \neq o_n(p_n(|\alpha_k|,\dots,|\alpha_1|), q_n(d_l,\dots,d_1)))$, M decrements $o_n(p_n(|\alpha_k|,\dots,|\alpha_1|), q_n(d_l,\dots,d_1))$ by one each time H meets the symbol "0" in the substring $0\sharp^{n_{e1}}0\sharp^{n_{e2}}\dots 0\sharp^{n_{em_e}}$ $(\triangleq v_e)$. In order to do so, M decrements d_1 (= the counting number of SC_1) by one each time H meets the symbol 0. In this case, for example, if $d_1 = 0$ when H meets the r-th 0 from the left in v_e, then

i. if $d_m \neq 0$ (where m is the smallest integer such that $d_m \neq 0$), then M decrements d_m by one instead of decrementing d_1 by one, and M sets $d_1 = d_2 = \dots = d_{m-1} = g(n,n,1)$ by using the (assumed) length n of $\sharp^{n_{el}}$'s in v_e (note that we assume that $n_{el} = n$ for each $1 \leq l \leq m_e$).

ii. if $d_1 = \dots = d_l = 0$ and $|\alpha_m| \neq 0$ (where m is the smallest integer such that $|\alpha_m| \neq 0$), then M erases the rightmost Z on C_m (i.e., $|\alpha_m| \leftarrow |\alpha_m| - 1$) by one instead of decrementing d_1 by one, and M sets $d_1 = d_2 = \dots = d_l = g(n,n,1)$ and $\alpha_1 = \alpha_2 = \dots = \alpha_{m-1} = Z^n$ by using the length n of $\sharp^{n_{el}}$'s in v_e.

M enters an accepting state only if H meets the last 0 in v_e with $|\alpha_1| = \dots = |\alpha_k| = d_1 = \dots = d_k = 0$ (i.e., $o_n(p_n(|\alpha_k|,\dots,|\alpha_1|), q_n(d_l,\dots,d_1)) = 0$) (resp., M enters an accepting state only if H meets the last "0" in v_e with $|\alpha_j| \neq 0$ for some $1 \leq j \leq k$ or with $d_i \neq 0$ for some $1 \leq i \leq l$ (i.e., $o_n(p_n(|\alpha_k|,\dots,|\alpha_1|), q_n(d_l,\dots,d_1)) \neq 0$) or H meets 0 in v_e after $|\alpha_1| = \dots = |\alpha_k| = d_1 = \dots = d_l = 0$).

[In order to decrement $d_i (1 \leq i \leq l)$ by one,

i. if $f_i = 1$ and $j_i \neq 0$, then M has only to set $f_i = 1$, and move the i-th stack-counter head one cell to the left (i.e., $j_i \leftarrow j_i - 1$),

ii. if $f_i = 1$ and $j_i = 0$, then M has only to set $f_i = 0$ with $j_i = 0$,

iii. if $f_i = 0$ and $j_i < |\beta_i|$, then M has only to set $f_i = 0$, and move the i-th stack-counter head one cell to the right (i.e., $j_i \leftarrow j_i + 1$), and

iv. if $f_i = 0$ and $j_i = |\beta_i| \neq 0$, then M has only to set $f_i = 1$, and erase the rightmost Z on SC_i (i.e., $|\beta_i| \leftarrow |\beta_i| - 1$ and $j_i \leftarrow j_i - 1$).

Note that if $f_i = 0$ and $j_i = |\beta_i| = 0$, then $d_i = g(|\beta_i|, j_i, f_i) = g(0,0,0) = 0$.]

The proof of (3): A 1nfacs($k, l+1$,real) M can accept $A(k,l)$ as follows. Let $C_1,\dots,C_k$ be the counters and $SC_1,\dots,SC_{l+1}$ be the stack-counters of M. For a presented input string, M checks the above two points (i) and (ii) in the proof of (1) and (2). That is, M checks by using SC_{l+1} wheter (i) above holds, and checks by using the same algorithm as in the proof of (1) whether (ii) holds.

The proof of (4) and (5): Suppose that there exists a 1afacs($k-1, l$,real) M which accepts $A(k,l)$ (resp., $A'(k,l)$). For each $n \geq 1$, let

$V(n) = \{\sharp^n 1^{s_1}\natural\dots\natural 1^{s_k}a^{t_1}b^{u_1}c_1\dots a^{t_l}b^{u_l}c_l h(n,m_1)\natural\dots\natural h(n,m_{L(n)}) | \forall j(1 \leq j \leq k)[0 \leq s_j \leq n]\ \&\ \forall i(1 \leq i \leq l)[0 \leq t_i \leq n, 0 \leq u_i \leq n - t_i, c_i \in \{0,1\}]\ \&\ \forall f(1 \leq f \leq L(n))[1 \leq m_f \leq L(n)]\ \&\ \exists e(1 \leq e \leq L(n))[m_e = o_n(p_n(n-s_k,\dots,n-s_1), q_n(g(n-t_l,n-t_l-u_l,c_l),\dots,g(n-t_1,n-t_1-u_1,c_1)))]$ (resp.,$\forall e(1 \leq e \leq L(n))[m_e \neq$

$o_n(p_n(n-s_k,\ldots,n-s_1),q_n(g(n-t_l,n-t_l-u_l,c_l),\ldots,g(n-t_1,n-t_1-u_1,c_1)))]])\} \subseteq A(k,l)$ (resp., $\subseteq A'(k,l)$), where $L(n) = \{(n+1)^k - 1\} \times \{g(n,n,1)+1\}^l + \{g(n,n,1)+1\}^l - 1 = (n+1)^k\{g(n,n,1)+1\}^l - 1$, and let $W(n) = \{h(n,m_1)\natural\ldots\natural h(n,m_{L(n)})|\forall i(1 \le i \le L(n))[1 \le m_i \le L(n)]\}$.
Note that for each $x = \sharp^n 1^{s_1}\natural\ldots\natural 1^{s_k}a^{t_1}b^{u_1}c_1\ldots a^{t_l}b^{u_l}c_l h(n,m_1)\natural\ldots\natural h(n,m_{L(n)})$ in $V(n)$, there exists an accepting computation tree of M on x which has the properties:

(i) for each computation path P from the root to a leaf, the length of P is $|x\$|$ and P represents a computation in which the input head moves one cell to the right in each step, and thus

(ii) for each node π labeled with an ID which M enters just after the input head has read the initial segment $\sharp^n 1^{s_1}\natural\ldots\natural 1^{s_k}a^{t_1}b^{u_1}c_1\ldots a^{t_l}b^{u_l}c_l$ of x, the length of each counter and stack-counter in $\ell(\pi)$ is bounded by $(k+2l+1)n+(k+l-1)$, since M operates in realtime and we assume that M can enter an accepting state only when falling off the right end marker \$.

For each storage state s of M and for each y in $W(n)$, let

$M_y(s)$

=1 if there exists an s-accepting computation tree of M on y such that for each computation path P from the root to a leaf, the length of P is $|y\$|$ and P represents a computation in which the input head moves one cell to the right in each step,

=0 otherwise.

For any two strings y, z in $W(n)$, we say that y and z are M-equivalent if $M_y(s) = M_z(s)$ for each storage state $s = (q,(\alpha_1,\ldots,\alpha_{k-1}),(\beta_1,\ldots,\beta_l),(j_1,\ldots,j_l))$ of M with $0 \le |\alpha_j| \le (k+2l+1)n+(k+l-1)$ $(1 \le j \le k-1)$ and $0 \le j_i \le |\beta_i| \le (k+2l+1)n+(k+l-1)$ $(1 \le i \le l)$. Clearly, M-equivalence is equivalence relation on strings in $W(n)$, and there are at most

$$E(n) = 2^{r\{(k+2l+1)n+(k+l)\}^{k+2l-1}}$$

M-equivalence classes, where r denotes the number of states of the finite control of M. We denote these M-equivalence classes by $\mathcal{C}_1,\mathcal{C}_2,\ldots,\mathcal{C}_{E(n)}$. For each $y = h(n,m_1)\natural\ldots\natural h(n,m_{L(n)})$ in $W(n)$, let $b(y) = \{m|\exists i(1 \le i \le L(n))[m = m_i]\}$. Furthermore, for each $n \ge 1$, let $R(n) = \{b(y)|y \in W(n)\}$. Then,

$$|R(n)| = \binom{L(n)}{1} + \binom{L(n)}{2} + \ldots + \binom{L(n)}{L(n)} = 2^{L(n)} - 1.$$

We can easily see that $\log E(n) = O(n^{k+2l-1})$ and $\log|R(n)| = O(n^{k+2l})$.[2] Thus, we have $|R(n)| > E(n)$ for large n. For such n, there must be some Q, Q' $(Q \ne Q')$ in $R(n)$ and some C_i $(1 \le i \le E(n))$ such that the following statement holds:
"There are two strings y, $z \in W(n)$ such that (a) $b(y) = Q \ne Q' = b(z)$, and (b) y, $z \in C_i$ (i.e., y and z are M-equivalent)."

Because of (a), we can, without loss of generality, assume that there is some positive integer m such that $1 \le m \le L(n)$ and $m \in b(y) - b(z)$. Clearly, there are some $s_1,s_2,\ldots,s_k$, and $(t_1,u_1,c_1),(t_2,u_2,c_2),\ldots,(t_l,u_l,c_l)$ such that $m = o_n(p_n(n-s_k,\ldots,n-s_1),q_n(g(n-t_l,n-t_l-u_l,c_l),\ldots,g(n-t_1,n-t_1-u_1,c_1)))$ and for such s_j $(1 \le j \le k)$ and (t_i,u_i,c_i) $(1 \le i \le l)$, it follows that

$y' = \sharp^n 1^{s_1}\natural 1^{s_2}\natural\ldots\natural 1^{s_k}a^{t_1}b^{u_1}c_1a^{t_2}b^{u_2}c_2\ldots a^{t_l}b^{u_l}c_l y \in A(k,l)$
(resp., $y' = \sharp^n 1^{s_1}\natural 1^{s_2}\natural\ldots\natural 1^{s_k}a^{t_1}b^{u_1}c_1a^{t_2}b^{u_2}c_2\ldots a^{t_l}b^{u_l}c_l z \in A'(k,l)$), and
$z' = \sharp^n 1^{s_1}\natural 1^{s_2}\natural\ldots\natural 1^{s_k}a^{t_1}b^{u_1}c_1a^{t_2}b^{u_2}c_2\ldots a^{t_l}b^{u_l}c_l z \notin A(k,l)$
(resp., $z' = \sharp^n 1^{s_1}\natural 1^{s_2}\natural\ldots\natural 1^{s_k}a^{t_1}b^{u_1}c_1a^{t_2}b^{u_2}c_2\ldots a^{t_l}b^{u_l}c_l y \notin A'(k,l)$).

But because of (b), y' is accepted by M iff z' is accepted by M, which is a contradiction. This completes the proof of (4) and (5).
The proof of (6) and (7): The proof is almost the same as that of (4) and (5) of the lemma, and so omitted here. □

We are now ready to have our main results.

Theorem 4.1. For each $k \ge 0$, $l \ge 0$ $((k,l) \ne (0,0))$,
(1) $1\mathrm{U}(k+1,l,\mathrm{real}) - 1\mathrm{A}(k,l,\mathrm{real}) \ne \phi$, and
(2) $1\mathrm{U}(k,l+1,\mathrm{real}) - 1\mathrm{A}(k,l,\mathrm{real}) \ne \phi$.

Theorem 4.2. For each $k \ge 0$, $l \ge 0$ $((k,l) \ne (0,0))$,
(1) $1\mathrm{N}(k+1,l+1,\mathrm{real}) - 1\mathrm{A}(k,l,\mathrm{real}) \ne \phi$,
(2) $1\mathrm{N}(k,l+2,\mathrm{real}) - 1\mathrm{A}(k,l,\mathrm{real}) \ne \phi$, and
(3) $1\mathrm{N}(k+3,l,\mathrm{real}) - 1\mathrm{A}(k,l,\mathrm{real}) \ne \phi$.

[2] For any set S, $|S|$ denotes the number of elements of S.

proof. (1) follows from Lemma 4.1 (3) and (4). (2) easily follows from (1) of the theorem. Since 1N($k, l+m$,real) $\subseteq$ 1N($k+2m, l$,real) for each $k \geq 0$, $l \geq 0$ and $m \geq 0$ ($(k,l,m) \neq (0,0,0)$) (Fact 2.1), (3) follows from (1) of the theorem. □

Book and Ginsburg [1] essentially showed that for each $k \geq 1$ and each $X \in \{N, D\}$,

$$1\mathrm{X}(k+1,0,\mathrm{real}) - 1\mathrm{X}(k,0,\mathrm{real}) \neq \phi, \text{ and } 1\mathrm{X}(0,k+1,\mathrm{real}) - 1\mathrm{X}(0,k,\mathrm{real}) \neq \phi.$$

Now, we strengthen this result, and show a relationship between the accepting powers of 1ufacs(k,l,real)'s, 1nfacs(k,l,real)'s and 1dfacs(k,l,real)'s.

Lemma 4.2. For each $k \geq 0$, $l \geq 0$ ($(k,l) \neq (0,0)$), let
$U(k,l) = \{0^{s_1}1^{t_1}0^{s_2}1^{t_2}\ldots 0^{s_l}1^{t_l}\sharp 1^{u_1}\sharp 1^{u_2}\sharp\ldots\sharp 1^{u_k} | \forall i(1 \leq i \leq l)[1 \leq t_i \leq s_i] \;\&\; \forall j(1 \leq j \leq k)[u_j \geq 1]\}$, and
$L(k,l) = \{wcw^R | w \in U(k,l)\}$.[3]
For each $k \geq 0$, $l \geq 0$ ($(k,l) \neq (0,0)$),
(1) $L(k,l) \in$ 1D(k,l,real) and
(2) $L(k,l)^c \in$ 1D(k,l,real),[4]
and for each $k \geq 1$, $l \geq 0$ ($(k-1,l) \neq (0,0)$),
(3) $L(k,l) \notin$ 1N($k-1,l$,real),
and for each $k \geq 0$, $l \geq 1$ and $m \geq 1$ ($l-m \geq 0$),
(4) $L(k,l) \notin$ 1N($k+2m-1, l-m$,real).

proof. By using the same technique as in the proof in [5], we can prove this lemma. So the proof is omitted here. □

Theorem 4.3. For each $k \geq 0$, $l \geq 0$ ($(k,l) \neq (0,0)$),
(1) 1D($k+1,l$,real) − 1N(k,l,real) $\neq \phi$, and
(2) 1D($k,l+1$,real) − 1N(k,l,real) $\neq \phi$.

We need the following lemma.

Lemma 4.3. For each $k \geq 0$, $l \geq 0$ ($(k,l) \neq (0,0)$), let co-1N(k,l,real) = $\{L^c | L \in$1N(k,l,real)$\}$.
Then, for each $k \geq 0$, $l \geq 0$ ($(k,l) \neq (0,0)$), 1U(k,l,real) = co-1N(k,l,real).

proof. By using the same technique as in the proof of Lemma 5.1 in [5], we can easily prove this lemma. □

Theorem 4.4. For each $k \geq 0$, $l \geq 0$ ($(k,l) \neq (0,0)$),
(1) 1D($k+1,l$,real) − 1U(k,l,real) $\neq \phi$, and
(2) 1D($k,l+1$,real) − 1U(k,l,real) $\neq \phi$.

proof. By Lemma 4.2 (3) and Lemma 4.3, we can show that $L^c(k,l) \notin$ 1U($k-1,l$,real) for each $k \geq 1$, $l \geq 0$ ($(k-1,l) \neq (0,0)$). The theorem follows from this fact and Lemma 4.2 (2). □

Corollary 4.1. For each $k \geq 0$, $l \geq 0$ ($(k,l) \neq (0,0)$), and each X$\in$$\{A,U,N,D\}$,
(1) 1X($k+1,l$,real) − 1X(k,l,real) $\neq \phi$, and
(2) 1X($k,l+1$,real) − 1X(k,l,real) $\neq \phi$.

Corollary 4.2. For each $k \geq 0$, $l \geq 0$ ($(k,l) \neq (0,0)$), and each X$\in$$\{A,U,N,D\}$,
(1) 1X(k,l,real) $\subsetneq$ 1X($k+1,l$,real), and
(2) 1X(k,l,real) $\subsetneq$ 1X($k,l+1$,real).

5 A Relationship between Counters and Stack-Counters

This section investigates a relationship between the accepting powers of realtime 1afacs's.
Book and Ginsburg [1] showed that for each $k \geq 1$,

$$1\mathrm{N}(0,k,\mathrm{real}) - 1\mathrm{N}(2k-1,0,\mathrm{real}) \neq \phi.$$

Yoshinaga, Inoue and Takanami [5] showed that for each $k \geq 1$,

$$1\mathrm{D}(0,k,\mathrm{real}) - 1\mathrm{U}(2k-1,0,\mathrm{real}) \neq \phi.$$

[3]For any string w, w^R denotes the reversal of w.
[4]For any language L, L^c denotes the complement of L.

We first show that a similar result holds for 1afacs's with only universal states and with only existential states.

Theorem 5.1. For each $k \geq 0$, $l \geq 0$ and $m \geq 1$,
(1) $1D(k, l+m, \text{real}) - 1N(k+2m-1, l, \text{real}) \neq \phi$, and
(2) $1D(k, l+m, \text{real}) - 1U(k+2m-1, l, \text{real}) \neq \phi$.

proof. (1) follows from Lemma 4.2 (1) and (4). By using the same technique as in the proof of Theorem 4.4, (2) follows from (1) of this theorem. □

Corollary 5.1. For each $k \geq 0$, $l \geq 0$ and $m \geq 1$, and each X∈{U,N,D}, $1X(k, l+m, \text{real}) - 1X(k+2m-1, l, \text{real}) \neq \phi$.

Inoue, Ito and Takanami [4] showed that for each $k \geq 1$,

$$1A(0, k, \text{real}) - 1A(k, 0, \text{real}) \neq \phi.$$

Yoshinaga, Inoue and Takanami [5] strengthened this result and showed that for each $k \geq 1$,

$1U(0, k, \text{real}) - 1A(k, 0, \text{real}) \neq \phi$, $1N(0, k, \text{real}) - 1A(k, 0, \text{real}) \neq \phi$ $(k \neq 2)$, and $1A(0, k, \text{real}) - 1A(2k-1, 0, \text{real}) \neq \phi$.

From Lemma 4.1, we now strengthen this result further.

Theorem 5.2. For each $k \geq 0$, $l \geq 0$ and $m \geq 1$,
(1) $1U(k, l+m, \text{real}) - 1A(k+2m-1, l, \text{real}) \neq \phi$, and
(2) $1N(k, l+m+1, \text{real}) - 1A(k+2m-1, l, \text{real}) \neq \phi$.

Theorem 5.3. For each $k \geq 0$, $l \geq 0$ and $m \geq 1$, $1A(k, l+m, \text{real}) - 1A(k+2m-1, l, \text{real}) \neq \phi$.

Remark Book and Ginsburg [1] showed that there are no i and j such that $1N(i, 0, \text{real}) = 1N(0, j, \text{real})$. It is shown in [5] that there are no i and j such that $1X(i, 0, \text{real}) = 1X(0, j, \text{real})$ for each X∈{U,D}. For each X∈{A,U,N,D}, if "$1X(k, l+1, \text{real}) \subsetneq 1X(k+2, l, \text{real})$ for each $k \geq 0$, $l \geq 0$" can be proved, then from the results in this paper, it follows that there are no pairs (i, j) and (k, l) such that $(i, j) \neq (k, l)$ and $1X(i, j, \text{real}) = 1X(k, l, \text{real})$.

6 Conclusion

In this paper, we presented several hierarchical results in the accepting powers of realtime 1afacs's. We conclude this paper by listing up some open problems.
(1) For each $k \geq 0$, $l \geq 0$ and each X∈{N,D},
$1X(k+1, l, \text{real}) - 1A(k, l, \text{real}) \neq \phi$? and $1X(k, l+1, \text{real}) - 1A(k, l, \text{real}) \neq \phi$?
(2) $1N(k, l+m, \text{real}) - 1A(k+2m-1, l, \text{real}) \neq \phi$ for each $k \geq 0$, $l \geq 0$ and $m \geq 1$?, and
(3) $1X(k, l+1, \text{real}) \subsetneq 1X(k+2, l, \text{real})$ for each $k \geq 0$, $l \geq 0$ and each X∈{A,U,N,D} ?

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